arXiv:quant-ph/0507029v3 25 Sep 2006

Entanglement measurement based on two-particle interference

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We propose a simple and realizable method using a two-particle interferometer for the experimental measurement of pairwise entanglement, assuming some prior knowledge about the quantum state. The basic idea is that the properties of the density matrix can be revealed by the single- and two-particle interference patterns. The scheme can easily be implemented with polarized entangled photons.

PACS numbers: 03.67.Mn, 03.65.Ud, 07.60.Ly, 42.65.Lm

Introduction Quantum entanglement is a kind of indispensable resource for exponential speedups of future quantum computers [1, 2] and long distance secret quantum communications [3, 4, 5]. However, for practical physical systems couplings with the external environment are unavoidable, which will result in various decoherence processes. Therefore, entanglement will be reduced consequentially [6, 7, 8], especially during creation in real experiments with noise and imperfections and distribution through a lossy channel. This makes it crucial to find efficient methods to detect entanglement, not only to test whether a given state — both pure states and mixed states — is entangled or not, but also to determine the degree of entanglement.

In the recent years, several methods for measurement of entanglement have been proposed. The most straightforward way is to reconstruct the quantum state fully through quantum tomography [9]. For systems of two qubits, it requires nine different measurement settings to determine 15 parameters which describe a general two-qubit state. However, not all these parameters are necessary for measurement of entanglement, assuming some prior knowledge of the density matrix. In [10], P. Horodecki and A. Ekert proposed a direct and efficient scheme which provides the estimation of the degree of entanglement of an unknown quantum state. This scheme is based on the quantum network which can evaluate certain nonlinear functionals of density matrixes. However, it requires the implementation of control unitary operations, which may not be accomplished very easily in real experiments. Another different approach is the method of entanglement witness operator W [11, 12]. For a density matrix ρ, which is entangled, the expectation value is negative $Tr(W\rho) < 0$, nevertheless the expectation value is positive $Tr(W\rho_{sep}) \geq 0$ for all separable states. With a few local measurements [13], one can obtain the expectation value of the entanglement witness and then detect the presence of entanglement. In the first experiment realization of entanglement witness [14], three different measurement settings are required for Werner states [13].

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In this paper, we introduce a new method for experimental measurement of entanglement using a two-particle interferometer, which is extended from standard one-particle interferometry. Lots of theoretical analyses and experiments have been carried out in this field [15, 16, 17]. Complementarity of one-particle and two-particle interference [18, 19] has been revealed. Here we reexamine the properties of one- and two-particle interference patterns of two particles in entangled states. It is showed that, assuming some prior knowledge about the entangled states, the degree of entanglement can be reflected by the properties of the interference patterns. Therefore, the degree of entanglement can be measured by study single- and joint-detection probabilities through a two-particle interferometer. This presents a new kind of utility of quantum interference in quantum information processing. Since techniques of two-particle interferometry have been developed very well, thus our scheme for measurement of entanglement is very simple and can be easily realized in practical experiments.

Two-particle interferometer A schematic two-particle interferometer [18, 19] is depicted in Fig. 1. The source S produce a pair of particles 1 and 2, which can be realized through a laser-pumped down-converting crystal [16]. Particle 1 propagates along paths A and/or A', passing through the passive lossless transducer T_1, and then emerges out in either U_1 or L_1. Similarly, particle 2 propagates along paths B and/or B', passing through the passive lossless transducer T_2, and then emerges out in either U_2 or L_2.

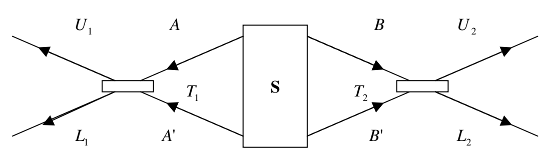

FIG. 1: Schematic two-particle interferometer. Two particles emit from the source S and pass through the passive lossless transducers T_1 and T_2. Then they are detected at the outports U_1, L_1 and U_2, L_2 respectively.

For simplicity, we denote $|0\rangle_1 = |A\rangle$, $|1\rangle_1 = |A'\rangle$ and $|0\rangle_2 = |B\rangle$, $|1\rangle_2 = |B'\rangle$ in the rest of this paper. Likewise, for the output Hilbert space $|0_o\rangle_i = |U_i\rangle$ and $|1_o\rangle_i =$

$|L_i\rangle$ $(i = 1, 2)$. The general entangled two-particle state ρ generated from the source S is in the Hilbert space spanned by the basis $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$. The transducer T_1 implements the unitary unimodular map for particle 1 as follows [19]

$$\begin{aligned} T_1|0'\rangle_1 &= ae^{i\phi_a}|0_o\rangle_1 + be^{i\phi_b}|1_o\rangle_1 \\ T_1|1'\rangle_1 &= -be^{-i\phi_b}|0_o\rangle_1 + ae^{-i\phi_a}|1_o\rangle_1 \end{aligned} \quad (1)$$

where $\{|0'\rangle_1, |1'\rangle_1\}$ is an orthonormal basis for the space spanned by $|0\rangle_1$ and $|1\rangle_1$, a and b are real numbers and satisfy $a^2 + b^2 = 1$. Similarly, the transducer T_2 implements the unitary unimodular map for particle 2 as [19]

$$\begin{aligned} T_2|0'\rangle_2 &= ce^{i\phi_c}|0_o\rangle_2 + de^{i\phi_d}|1_o\rangle_2 \\ T_2|1'\rangle_2 &= -de^{-i\phi_d}|0_o\rangle_2 + ce^{-i\phi_c}|1_o\rangle_2 \end{aligned} \quad (2)$$

where $\{|0'\rangle_2, |1'\rangle_2\}$ is an orthonormal basis for the space spanned by $|0\rangle_2$ and $|1\rangle_2$, c and b are real numbers and satisfy $c^2 + d^2 = 1$. Then after passing through two transducers T_1 and T_2, the state of particle 1 and particle 2 is transformed into $\rho' = (T_1 \bigotimes T_2)\rho(T_1 \bigotimes T_2)^\dagger$. Here ρ' is in the Hilbert space spanned by the basis $\{|0_o\rangle_1|0_o\rangle_2, |0_o\rangle_1|1_o\rangle_2, |1_o\rangle_1|0_o\rangle_2, |1_o\rangle_1|1_o\rangle_2\}$. Now we can measure the single detection probability $P(U_1) = \langle 0_o|_2\langle 0_o|_1\rho'|0_o\rangle_1|0_o\rangle_2 + \langle 1_o|_2\langle 0_o|_1\rho'|0_o\rangle_1|1_o\rangle_2$ and the probability of joint output $P(U_1U_2) = \langle 0_o|_2\langle 0_o|_1\rho'|0_o\rangle_1|0_o\rangle_2$. Other analogous probabilities $P(U_1L_2)$, $P(L_1U_2)$, $P(L_1L_2)$ and $P(U_2)$, $P(L_1)$, $P(L_2)$ can be written in a similar way.

One-particle interference We start by investigating the single-particle fringe visibility. Suppose particle 1 is prepared in an arbitrary two-level quantum state ρ. For our purpose ρ is expressed in the basis $\{|0\rangle_1, |1\rangle_1\}$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \quad (3)$$

When particle 1 enters and exits the transducer T_1, it experiences the unitary map induced by T_1 and its state changes into $\rho' = T_1\rho T_1^\dagger$ consequently. We express ρ' in the basis $\{|0_o\rangle_1, |1_o\rangle_1\}$ and then the single detection probability can be obtained straightforwardly $P(U_1) = \rho'_{0_o0_o}$, that is

$$P(U_1) = a^2\rho_{00} + b^2\rho_{11} - abe^{i\phi_1}\rho_{01} - abe^{-i\phi_1}\rho_{10} \quad (4)$$

where $\phi_1 = \phi_a + \phi_b$. We now turn to the single-particle fringe visibility V_1, which is defined as follows [18]

$$V_1 = \frac{[P(U_1)]_{max} - [P(U_1)]_{min}}{[P(U_1)]_{max} + [P(U_1)]_{min}} \quad (5)$$

To find the extreme of $P(U_1)$ we first note that $P(U_1) = a^2\rho_{00} + b^2\rho_{11} - 2abRe[\rho_{01}e^{i\phi_1}]$. Then for some fixed a and b, the extreme are achieved $[P(U_1)]_{max} = a^2\rho_{00} + b^2\rho_{11} + 2|ab\rho_{01}|$ and $[P(U_1)]_{in} = a^2\rho_{00} + b^2\rho_{11} - 2|ab\rho_{01}|$ by choosing appropriate phase parameters ϕ_a and ϕ_b. Assuming $|a| = \cos(\mu/2)$ and $|b| = \sin(\mu/2)$ with $0 \le \mu \le \pi$, we can get

$$\begin{aligned} [P(U_1)]_{max} &= \frac{1}{2} + \frac{\rho_{00} - \rho_{11}}{2}\cos\mu + |\rho_{01}|\sin\mu \\ &= \frac{1}{2} + \frac{\sqrt{I_1}}{2}\sin(\mu + \mu') \end{aligned} \quad (6)$$

where $I_1 = (\rho_{00} - \rho_{11})^2 + 4|\rho_{01}|^2$ and μ' satisfies that $\sin\mu' = (\rho_{00} - \rho_{11})/\sqrt{I_1}$ and $\cos\mu' = 2|\rho_{01}|/\sqrt{I_1}$. Therefore, it can be easily seen that

$$[P(U_1)]_{max} = \frac{1 + \sqrt{I_1}}{2} \quad (7)$$

The minimum value $[P(U_1)]_{min}$ can be obtained in a similar way

$$[P(U_1)]_{min} = \frac{1 - \sqrt{I_1}}{2} \quad (8)$$

Now according to the definition in Eq. (5), the square of the single-particle fringe visibility for particle 1 is

$$V_1^2 = I_1 \quad (9)$$

Before proceeding to the next part, it is worth for us to discuss the physical meaning of the above result in Eq. (9). We find that $I_1 = \rho_{00}^2 + \rho_{11}^2 - 2\rho_{00}\rho_{11} + 4|\rho_{01}|^2 = 2(\rho_{00}^2 + \rho_{11}^2 + 2|\rho_{01}|^2) - 1$. Here we have used $\rho_{00} + \rho_{11} = 1$. Note that $Tr\rho^2 = \rho_{00}^2 + \rho_{11}^2 + 2|\rho_{01}|^2$, therefore we have $I_1 = 2Tr\rho^2 - 1$, which is just the total information content in a two-state quantum system [20]. The total information is defined as the sum over a complete set of mutually complementary measurements. For a spin-1/2 particle in the state ρ, a complete set of mutually complementary experiments consists of three measurements. The relation, depicted above, shows that the single-particle fringe visibility V_1 reflects the total information content in the quantum state ρ. Conversely, through one-particle interferometry the total information content and then the purity of the quantum state can be determined too.

Scheme for measurement of entanglement In the following we consider a special class of entangled two-particle states of the form

$$\rho = p|\psi\rangle\langle\psi| + (1-p)\frac{I}{4}, \quad 0 \le p \le 1 \quad (10)$$

where $|\psi\rangle$ is an arbitrary pure state $|\psi\rangle = \lambda_1|0\rangle_1|0\rangle_2 + \lambda_2|0\rangle_1|1\rangle_2 + \lambda_3|1\rangle_1|0\rangle_2 + \lambda_4|1\rangle_1|1\rangle_2$ with the normalization condition $|\lambda_1|^2 + |\lambda_2|^2 + |\lambda_3|^2 + |\lambda_4|^2 = 1$. According to the Schmidt decomposition theorem, $|\psi\rangle$ can be expressed as $|\psi\rangle = \alpha|0'\rangle_1|1'\rangle_2 + \beta|1'\rangle_1|0'\rangle_2$, where $|\alpha|^2 + |\beta|^2 = 1$. Without loss of generality, we can suppose $|\alpha|^2 \ge |\beta|^2$ here. When $|\psi\rangle = |\Psi_-\rangle$ the above entangled states is just the Werner states [21]. The state ρ given in Eq. (10) exists in many practical realizations. The pure part $|\psi\rangle\langle\psi|$ is the ideal entangled state to be produced through real experiments with noise and imperfections or to be distributed between two distant parties over a lossy channel. And the maximally chaotic part $I/4$ is induced by

the decoherence processes. When p is larger than some critical value, the above state ρ is entangled. Otherwise, it is separable. The degree of entanglement is not only dependent on p but also determined by the pure state $|\psi\rangle$.

In the basis $\{|0'\rangle_1|0'\rangle_2, |0'\rangle_1|1'\rangle_2, |1'\rangle_1|0'\rangle_2, |1'\rangle_1|1'\rangle_2\}$ the state ρ can be expressed as

$$\rho = \begin{pmatrix} w & 0 & 0 & 0 \\ 0 & x & z & 0 \\ 0 & z^* & y & 0 \\ 0 & 0 & 0 & w \end{pmatrix} \tag{11}$$

where $w = (1-p)/4$, $x = (1-p)/4 + p|\alpha|^2$, $y = (1-p)/4 + p|\beta|^2$ and $z = p\alpha\beta^*$. We denote the time-reversed matrix of ρ as $\widetilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. After straightforward calculations, the square roots of four eigenvalues of $\rho\widetilde{\rho}$ can be expressed as $\{w, w, \sqrt{xy} - |z|, \sqrt{xy} + |z|\}$. Therefore, the entanglement of ρ, measured by concurrence [22], is

$$C = 2\max\{p|\alpha\beta| - \frac{1-p}{4}, 0\} \tag{12}$$

Our main goal is to measure C from the fringes of single- and two-particle interference. The visibility of single-particle fringe has been analyzed above. In order to achieve the goal of entanglement measurement, we need to investigate the property of two-particle fringe.

In the two-particle interferometer, after the two particles passing through the passive and lossless transducers T_1 and T_2, the entangled state ρ changes into $\rho' = (T_1 \bigotimes T_2)\rho(T_1 \bigotimes T_2)^\dagger$. Therefore, the probability of joint detection by ideals detectors placed at the output forts U_1 and U_2 is $P(U_1U_2) = \langle 0_o|_2\langle 0_o|_1\rho'|0_o\rangle_1|0_o\rangle_2$, which can be calculated as follows

$$\begin{aligned} P(U_1U_2) &= a^2(c^2w + d^2x) + b^2(c^2y + d^2w) \\ &\quad + abcd(p\alpha\beta^* e^{i\phi} + p\alpha^*\beta e^{-i\phi}) \end{aligned} \tag{13}$$

where a, b, c, d and $\phi = \phi_a + \phi_b - \phi_c - \phi_d$ are parameters describing the transducers T_1 and T_2. Note that $P(U_1U_2) = a^2(c^2w + d^2x) + b^2(c^2y + d^2w) + 2pabcdRe[\alpha\beta^* e^{i\phi}]$. Then when the parameters a, b, c, d are fixed, the maximum probability of joint output are achieved $[P(U_1U_2)]_{max} = a^2(c^2w+d^2x)+b^2(c^2y+d^2w)+2pabcd|\alpha\beta|$ by choosing appropriate phase parameters ϕ_a, ϕ_b and ϕ_c, ϕ_d. Assuming $|a| = \cos(\mu/2)$, $|b| = \sin(\mu/2)$ and $|c| = \cos(\upsilon/2)$, $|d| = \sin(\upsilon/2)$ with $0 \le \mu, : \upsilon \le \pi$, we can get

$$\begin{aligned} [P(U_1U_2)]_{max} &= \frac{1}{4} + m(\cos\mu - \cos\upsilon) - \frac{p}{4}\cos\mu\cos\upsilon \\ &\quad + n\sin\mu\sin\upsilon \end{aligned} \tag{14}$$

where $m = p(|\alpha|^2 - |\beta|^2)/4, n = p|\alpha\beta|/2$.

Since $|\alpha\beta| \le (|\alpha|^2 + |\beta|^2)/2 = 1/2$, thus $n \le p/4$. Therefore it follows that

$$\begin{aligned} [P(U_1U_2)]_{max} &= \frac{1}{4} + m(\cos\mu - \cos\upsilon) - \frac{p}{4}\cos(\mu+\upsilon) \\ &- (\frac{p}{4} - n)\sin\mu\sin\upsilon \le \frac{p+1}{4} + 2m \\ &= \frac{p+1}{4} + \frac{p}{2}(|\alpha|^2 - |\beta|^2) \end{aligned} \tag{15}$$

The equation is satisfied when $\mu = 0$ and $\upsilon = \pi$. For the sake of discussion later, we denote $P_{12} = [P(U_1U_2)]_{max} = (p+1)/4 + p(|\alpha|^2 - |\beta|^2)/2$.

In addition, for the state ρ given in Eq. (10), the single-particle fringe visibility, according to Eq. (9), is $V_1^2 = I_1 = 2Tr\rho_1^2$. Here $\rho_1 = Tr_2\rho$ is the reduced density matrix of particle 1. Therefore, the single-particle fringe visibility can be written straightforwardly

$$V_1^2 = p^2(1 - 4|\alpha|^2|\beta|^2) \tag{16}$$

Note that $|\alpha|^2 + |\beta|^2 = 1$, from Eq. (15) and (16) we can get

$$P_{12} = (p+1)/4 + V_1/2 \tag{17}$$

Therefore, the parameter p can be evaluate as

$$p = 4P_{12} - 2V_1 - 1 \tag{18}$$

By Eq. (12), (18) and note that $2p|\alpha\beta| = \sqrt{p^2 - V_1^2}$, the amount of entanglement in the state ρ is

$$\begin{aligned} C &= \max\{2P_{12} - V_1 - 1 \\ &\quad + \sqrt{(4P_{12} - 3V_1 - 1)(4P_{12} - V_1 - 1)}, 0\} \end{aligned} \tag{19}$$

It can be seen that in order to determine the amount of entanglement in the state ρ give in Eq. (12), we only need to measure the single-particle fringe visibility V_1 and the maximum probability of joint output P_{12}. And then based on Eq. (19), the value $2P_{12} - V_1 - 1 + \sqrt{(4P_{12} - 3V_1 - 1)(4P_{12} - V_1 - 1)}$ can be evaluated. If this value is negative then the state ρ is separable. Otherwise, this value is just the concurrence of ρ to be measured. That is we have proposed a simple method based on two-particle interference for measurement of entanglement, assuming some prior knowledge about the quantum state. For the Werner class of mixed states, only two quantities V_1 and P_{12} are required to be measured in application of our scheme.

Discussions From the concurrence expression in Eq. (19), we can see a natural requirement for the single-particle fringe visibility V_1 and the maximum probability of joint output P_{12} that is $4P_{12} - 3V_1 - 1 \ge 0$. From Eq. (17), we can see that this requires that $4P_{12} - 3V_1 - 1 = p - V_1 \ge 0$. Obviously, the above condition can be satisfied according to the expression for the single-particle fringe visibility V_1 in Eq. (16). It should be pointed out that the above requirement $4P_{12} - 3V_1 - 1 \ge 0$ is satisfied for the Werner class of mixed states given in

Eq. (10). However, it is not an universal requirement for any general two-particle states.

An illustrative demonstration of our scheme for entanglement measurement can be implemented with entangled polarized photons in Werner states $\rho = p|\Psi_-\rangle\langle\Psi_-| + (1-p)\frac{I}{4}$, $0 \leq p \leq 1$, produced via spontaneous parametric down-conversion (SPDC) [23, 24, 25]. The combination of two half-wave plate (HWP) and one quarter-wave plate (QWP) yields an arbitrary unitary rotation on polarized photons [26], which functions as the passive lossless transducers T_1 and T_2. Together with some linear optical instruments, such as polarizing beam splitter (PBS), the single-particle fringe visibility V_1 and the maximum joint probability P_{12} can be obtained straightforwardly. Therefore, the degree of entanglement in the Werner states can be measured successfully based on our scheme.

The idea of this paper can be easily applied for the detection of entanglement in some other special class of quantum states other than the Werner class of mixed states given in Eq. (10). For example, the initial states for entanglement purification [27] and the Gisin mixed states [11, 28]. In fact, the interferometric method of entanglement measurement is applicable to those mixed states that need only two independent parameters to characterize the degree of entanglement. For example, these two independent parameters for Werner states are p and α, for Gisin mixed states are a and x in [11, 28]. The measure of the single-particle fringe visibility V_1 and the maximum joint probability P_{12} will present two independent equations for these two independent parameters. Therefore, the degree of entanglement can be measured through the interferometric method. Although, we need some prior knowledge about the state and the relation between entanglement and V_1, P_{12} maybe different for different class of mixed states. However, our scheme is suitable for many practical situations [24, 25, 27]. Furthermore, we can generalize our scheme to measure the true $3-$qubit entanglement [29] of $3-$qubit pure states. Similarly, only two-particle interferometer is required, which will be discussed detailedly in our future work. In addition, whether we can make use of multi-particle interferometry to measure the degree of entanglement in general multi-qubit states is still an open and interesting question.

Conclusions In summary, we analyze the relation between the interference patterns and the properties of the quantum states. It is shown that the single-particle fringe visibility is just related to the information content in the quantum state of the particle. This result also links the purity of the state with the single-particle fringe visibility. In addition, we propose a simple scheme for measurement of entanglement using a two-particle interferometer, assuming some prior knowledge about the quantum state. The scheme is applicable for several special class of mixed states, e.g Werner class of mixed states and Gisin mixed states. An illustrative demonstration of our scheme can be easily implemented with polarized photon pairs produced via spontaneous parametric down-conversion. The optical instruments and quantum interferometric techniques required for practical experiments are all achievable.

Acknowledgments We thank Dr. B. H. Liu for helpful discussions. This work was funded by National Fundamental Research Program (2001CB309300) and NCET-04-0587, the Innovation funds from Chinese Academy of Sciences, and National Natural Science Foundation of China (Grant No. 60121503, 10574126).

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